Gelfand-Tzetlin bases, Jucy's Murphys elements and the relationship to association schemes

Ken Johnson, Penn State Abington

The construction of the character theory for a finite group (or quasigroup) G is via the association scheme which comes from the class algebra of the group (or quasigroup). The representation theory of the symmetric group S_n goes back to early days and connects with much combinatorics. However, the original way of doing things introduces ad. hoc. constructions.

Vershik and Oukunkov have produced a natural way of getting this representation theory, using the "Jucys-Murphy" elements of the group algebra of S_n. These elements are defined in terms of sums of involutions as follows:

$$X_0 = 0,\ X_1 = (12),\ X_i = (1i) + (2i) + ::: + ((i-1)i), ...$$

Although the elements $\{X_i\}$ are not in the center of $\mathbb{C}S_n$ they form a commutative algebra. The Jucys-Murphy elements give an example of a Gelfand-Tzetlin basis, an important concept in Lie group theory. One interesting fact is that the number of Jucys-Murphy elements of S_n is the same as the degree of the total character of S_n which is $\sum_{\chi \in Irr(S_n)} \chi(e)$. I will explore the relationship between the two approaches and generalizations to association schemes. The question of whether families of loops or quasigroups exist with such bases may be interesting.