Random walks which converge after a finite number of steps

Ken Johnson, Penn State Abington

A random walk on a finite group G associated to a probability p may be defines as follows. The states for the walk are the elements of G and if the walk is at an element h it moves to the element hg with probability $p(g)$. After n steps of the walk, the probability of being at a given element g is $p^{*n}(g)$ where the convolution

$$p_1 * p_2(g) = \sum_{h \in G} p_1(gh^{-1}) p_2(h).$$

and $p^{*n}(g) = p * p * ... * p$ (n times). The uniform probability U on G is defined by $U(g) = 1/|G|$ for all g in G.

The following question appeared in a paper by Zhmud and Vishnevetskiy

What are the conditions on G and p such that

$$p^{*n} = U \tag{1}$$

for some finite positive integer n?

They define $\Omega(G)$ to be the set of probabilities p such that (1) holds. Their results may be summarized as follows.

(a) The groups for which $\Omega(G) = U$ are either abelian groups or of the form $A \times Q_8$ where A is an elementary abelian 2-group.

(b) For any $p \in \Omega(G)$, $p^{*r} = U$ where r is the maximal degree of an irreducible representation of G.

(c) If $f \in \Omega(G)$ and f is constant on conjugacy classes then $f = U$.

I will explain an approach via group matrices and relate the result to a result of Taussky which gives a completely different condition on a group to be in the class described in (a). It may be that the question finding $\Omega(Q)$ for loops may be interesting-for example it would appear that if Q is the Octonion loop $\Omega(Q) = 0$.