

# GROUP-CASE ASSOCIATION SCHEMES AND THEIR CLASSIFICATION

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ABSTRACT. Many commutative association schemes are coming from finite transitive permutation groups. Let  $G$  be a finite group acting transitively on a finite set  $X$  (each group element  $g \in G$  permuting each element  $x \in X$  to element  $x^g$ ). This action is naturally extended to move each pair  $(x, y)$  to  $(x^g, y^g)$  in  $X \times X$  by  $g \in G$ . Let  $R_0, R_1, \dots, R_d$  be the orbits of the action of  $G$  on  $X \times X$  (which are often referred to as the 2-orbits of the permutation group  $G$  on  $X$ ). Then the set  $X$  and the set of 2-orbits,  $(X, \{R_i\}_{0 \leq i \leq d})$ , forms an association scheme, called a group-case (or Schurian) association scheme.

Under the action of  $G$  on  $X$ , let  $H$  be the stabilizer of a point  $x \in X$ . Let  $\theta$  be the permutation character of  $G$  on  $X$ ; so  $\theta$  is equivalent to the induced character of the identity character of  $H$ . If  $\theta$  is multiplicity-free; i.e.,  $\theta$  is decomposed into a direct sum of inequivalent irreducible characters, then the centralizer algebra (or the Hecke algebra) of the permutation group is commutative and the associated association scheme is commutative. That is, whenever we have such a pair  $(G, H)$  of groups, so called a Gelfand pair, we obtain a commutative association scheme.

It is well known that knowing the character table of a commutative scheme associated to a Gelfand pair is equivalent to knowing the zonal spherical functions of the permutation group. Many examples of Gelfand pairs and the character tables of the associated commutative association schemes have been studied by many researchers including, Bannai, Cohen, Gow, Henderson, Inglis, Klyachko, Kawanaka, Lawther, Liebeck, Lusztig, and Saxl. In this talk, we discuss some known examples of classical groups as their sources of Gelfand pairs, associated group-case association schemes, their character tables, and related problems as time permits.