

## MATH 618 SPRING 2016 GRADED HOMEWORK #0

Write clearly. Credit is given for the best three answers.

- (1) Let  $C^1(\mathbb{R})$  be the real vector space of differentiable real-valued functions on  $\mathbb{R}$ . Consider the exponential function

$$\exp: \mathbb{R} \rightarrow \mathbb{R}; x \mapsto e^x.$$

- (a) Express  $\exp$  as a sum of an even function  $f^+$  and an odd function  $f^-$ .  
(b) What are the standard names for the functions  $f^+$  and  $f^-$ ?
- (2) The matrices

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

are the respective bodies of character tables of abelian groups. Identify the abelian groups for each, and justify your answer.

- (3) Consider two representations  $\rho_i: G \rightarrow \mathbf{GL}(W_i)$  of a group  $G$ , for  $i = 1, 2$ .
- (a) Show that  $V = W_1 \oplus W_2$  and  $V' = W_2 \oplus W_1$  are respective representation spaces for  $G$ .  
(b) Show explicitly that the two representations  $\rho: G \rightarrow \mathbf{GL}(V)$  and  $\rho': G \rightarrow \mathbf{GL}(V')$  of  $G$  given in (a) are similar.
- (4) Consider  $1 < n \in \mathbb{Z}$ . Determine

$$\sum_{j=0}^{n-1} \exp\left(\frac{2\pi i j}{n}\right),$$

justifying your answer.