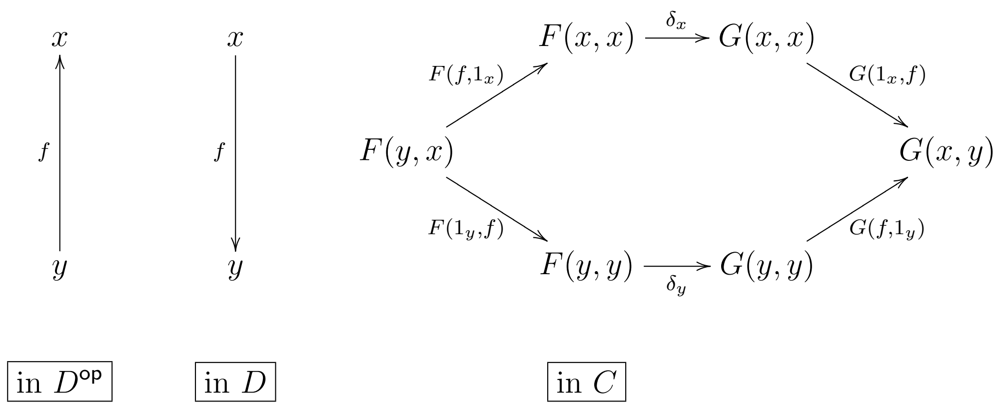
41. Dinatural transformations and power objects

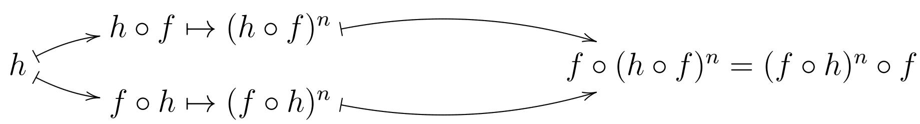
Given graph maps $F, G\colon D^{\mathsf{op}} \times D \to C$ for graph D and category C, a **dinatural transformation** $\delta\colon F \Rightarrow G$ is a "vector" $(\delta_x \mid x \in D_0)$ of **components** $\delta_x\colon F(x,x) \to G(x,x)$ in C_1 such that,

for all $f\colon x \to y$ in D_1, the hexagon of the **dinaturality diagram**



commutes in the category C.

Example: $\delta^n_x\colon C(x,x) \to C(x,x); k \mapsto k^n$ — **Church numeral** n.



Example: For $\mathbf{C}$ with (finite limits and) a subobject classifier Ω,

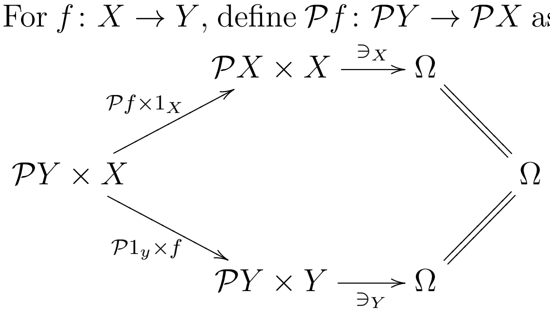
$$\mathbf{Sub_C}(X \times Y) \cong \mathbf{C}(X \times Y, \Omega) \cong \mathbf{C}(X, \mathcal{P}Y)$$

defines the **power object** $\mathcal{P}Y$ of an object Y.

In $\mathbf{C}(\mathcal{P}Y \times Y, \Omega) \cong \mathbf{C}(\mathcal{P}Y, \mathcal{P}Y)$, suppose $\ni_Y \mapsto 1_{\mathcal{P}Y}$.

In **Set**, have $\ni_Y$ as characteristic function of $\{(S,y) \in \mathcal{P}Y \times Y \mid S \ni y\}$.

For $f\colon X \to Y$, define $\mathcal{P}f\colon \mathcal{P}Y \to \mathcal{P}X$ as the unique morphism making



commute.

So dinatural $\boxed{\ni\colon \mathcal{P} \times 1_{\mathbf{C}} \Rightarrow \Delta\Omega}$ for $\mathcal{P}\colon \mathbf{C}^{\mathsf{op}} \to \mathbf{C}$.