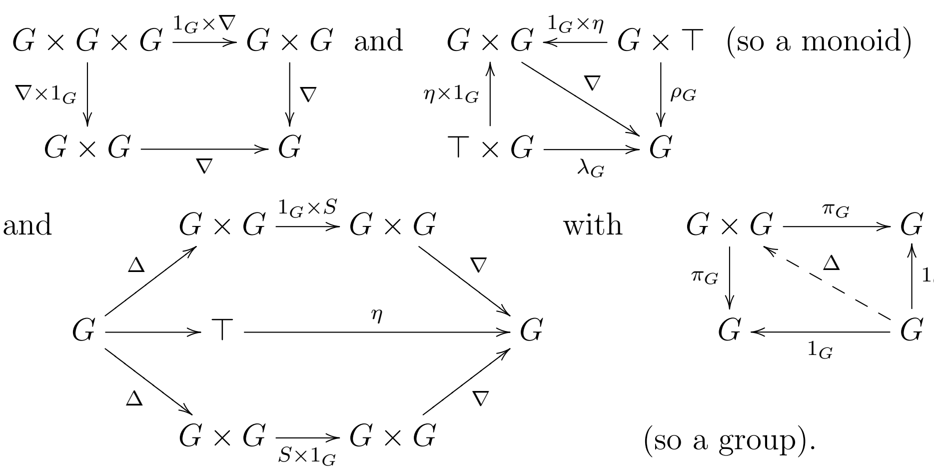
13. Groups in categories

Group $(G, \nabla\colon G \times G \to G, S\colon G \to G, \eta\colon \top \to G)$ in **Set**, satisfying:



Group in a Cartesian monoidal category: interprets the diagrams.

Example: A **topological group** is a group in the category **Top** of continuous maps between topological spaces.

Example: The additive group functor
$G_a\colon K \mapsto (K, +, -, 0)$ is a group in $\mathbf{Grp}^{\mathbf{CRing}}$.

Example: The multiplicative group or group-of-units functor
$G_m\colon K \mapsto (K^*, \cdot, ^{-1}, 1)$ is a group in $\mathbf{Grp}^{\mathbf{CRing}}$.

Example: The p-th roots of unity functor
$\mu_p\colon K \mapsto (\{k \in K \mid k^p = 1\}, \cdot, ^{p-1}, 1)$ is a group in $\mathbf{Grp}^{\mathbf{CRing}}$.

Example: $(\mathsf{SL}_2, \nabla, S, \eta)$ as a group in $\mathbf{Grp}^{\mathbf{CRing}}$:

$$\mathsf{SL}_2\colon \mathbf{CRing} \to \mathbf{Grp}; K \mapsto \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \;\middle|\; a, b, c, d \in K\ ,\ ad - bc = 1 \right\}.$$

$$\nabla_K\colon \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \right) \mapsto \begin{bmatrix} aa' + bc' & ab' + bd' \\ ca' + dc' & cb' + dd' \end{bmatrix},$$

$$S_K\colon \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mapsto \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ and } \eta_K\colon 1 \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$