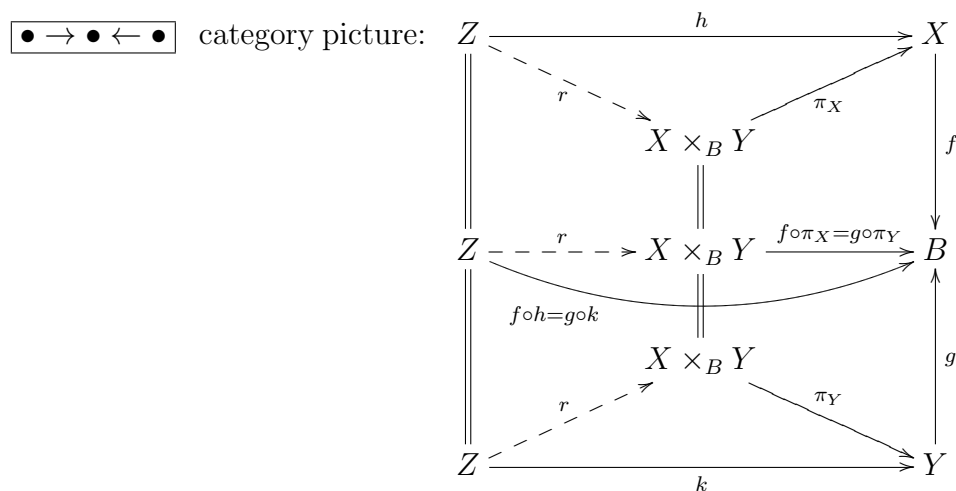
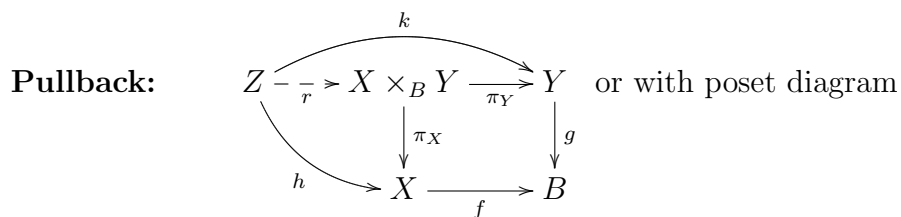
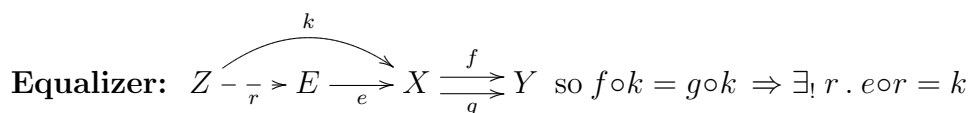


## 9. MORE LIMITS AND COLIMITS

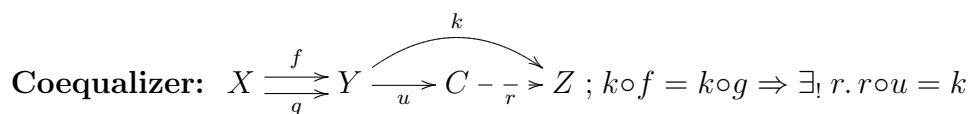


**Ex:** Domain of category composition is pullback of  $C_1 \xrightarrow{\partial_0} C_0 \xleftarrow{\partial_1} C_1$ .

**Pushout** is the dual of a pullback.



In  $\mathcal{L}$ ,  $E = \text{Ker}(f - g) \xrightarrow{e} X$ . In **Set**,  $E = \{x \in X \mid fx = gx\} \xrightarrow{e} X$ .



In **Set**,  $C$  is quotient of  $Y$  by equiv. rel'n. gen. by  $\{(fx, gx) \mid x \in X\}$ .

In  $\mathcal{L}$ ,  $u$  projects from  $Y$  to  $C = \text{Coker}(f - g) := Y/\text{Im}(f - g)$ .

**Extended First Isomorphism Theorem** in  $\mathcal{L}$  is the exact sequence

$$0 \longrightarrow \text{Ker } f \longrightarrow X \xrightarrow{f} Y \longrightarrow \text{Coker } f \longrightarrow 0$$

where **exact** means  $\text{Im } g_1 = \text{Ker } g_2$  for each  $\xrightarrow{g_1} \bullet \xrightarrow{g_2}$ .

Similar in **Ab**,  ${}_R\mathbf{Mod}$ ,  $\mathbf{Mod}_R$ ,  $\mathbf{Mod}_K$  (commutative unital ring  $K$ ), or any **abelian category**  $A$  where each  $A(X, Y)$  is an abelian group.