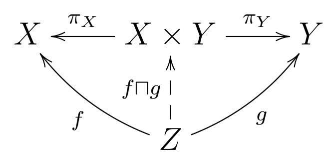
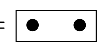
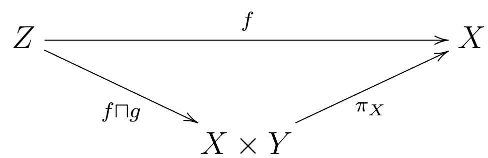
8. Products and coproducts

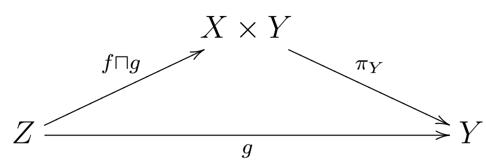
Product $X \times Y = \{(x, y) \mid x \in X\ ,\ y \in Y\}$ of sets X, Y:



Universality property: $\forall\, Z \in \mathbf{Set}_0$, "solid"↓ implies ↓"dashed"
bijection $\mathbf{Set}(Z, X) \times \mathbf{Set}(Z, Y) \to \mathbf{Set}(Z, X \times Y); (f, g) \mapsto f \sqcap g$
with $f = \pi_X \circ (f \sqcap g)$ and $g = \pi_Y \circ (f \sqcap g)$. Thus $f \sqcap g \colon z \mapsto (fz, gz)$.

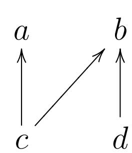
Picture in $\mathbf{Set}^2$ for discrete "two spot" diagram $2 =$  :





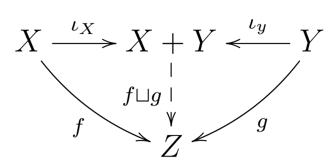
Examples: Product in **Set** carries products in **Grp**, **Ring**, **Mon**, etc.

Example: Product in a poset category is a **greatest lower bound**.



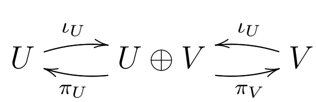
$a \times b = c$ exists,

but $c \times d$ does not.

Coproduct in C is the product in C^{op}: 

Example: Coproduct in **Set** is the disjont union.

Example: Coproduct in a poset category is a **least upper bound**.

Biproduct  in $\mathcal{L}$ is product and coproduct.