1. Directed graphs or quivers

What is category theory?

- Graph theory on steroids
- Comic book mathematics
- Abstract nonsense
- The secret dictionary

Sets and classes: For $S = \{X \mid X \notin X\}$, have $S \in S \Leftrightarrow S \notin S$

Directed graph or **quiver:** $C = (C_0, C_1, \partial_0 \colon C_1 \to C_0, \partial_1 \colon C_1 \to C_0)$
Class C_0 of objects, vertices, points, ...
Class C_1 of morphisms, (directed) edges, arrows, ...
For $x, y \in C_0$, write $C(x,y) := \{f \in C_1 \mid \partial_0 f = x, \partial_1 f = y\}$

tail, domain $\longrightarrow$ $\qquad \partial_0 f \xrightarrow{f \in C_1} \partial_1 f \qquad$ $\longleftarrow$ head, codomain

Opposite or **dual** graph of $C = (C_0, C_1, \partial_0, \partial_1)$ is $C^{\mathsf{op}} = (C_0, C_1, \partial_1, \partial_0)$

Graph homomorphism $F \colon D \to C$
has **object part** $F_0 \colon D_0 \to C_0$
and **morphism part** $F_1 \colon D_1 \to C_1$
with $\partial_i \circ F_1(f) = F_0 \circ \partial_i(f)$ for $i = 0, 1$.

Graph isomorphism has bijective object and morphism parts.

Poset $(X, \leq)$: set X with reflexive, antisymmetric, transitive **order** $\leq$

Hasse diagram of poset $(X, \leq)$: $x \to y$ if y **covers** x, i.e.,
$x \neq y$ and $[x, y] = \{x, y\}$, so $x \leq z \leq y \;\Rightarrow\; z = x$ or $z = y$.

Hasse diagram of $(\mathbb{N}, \leq)$ is $0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 3 \longrightarrow \ldots$

Hasse diagram of $(\{1, 2, 3, 6\}, \mid)$ is

$$\begin{array}{ccc} 3 & \longrightarrow & 6 \\ \uparrow & & \uparrow \\ 1 & \longrightarrow & 2 \end{array}$$