MATH 504 FALL 2008 PRACTICE MID-TERM

Hand in 3 answers for grading. Each question is worth $8\frac{1}{3}$ points.

(1) Suppose that a function $f : A \to B$ has non-empty domain. Show that there is a function $g : B \to A$ with $fgf = f$.

(2) Let A be a set. Prove that the direct power A^n is isomorphic to $\underline{\underline{\mathrm{Set}}}(\{1, 2, \ldots, n\}, A)$ for each positive integer n.

(3) Let α be a reflexive and transitive relation on a set A. Define a relation β on A by

$$x \ \beta \ y \qquad \Leftrightarrow \qquad (x \ \alpha \ y \text{ and } y \ \alpha \ x).$$

   (a) Prove that β is an equivalence relation on A.

   (b) Prove that

$$x^\beta \leq y^\beta \qquad \Leftrightarrow \qquad x \ \alpha \ y$$

   yields a well-defined order relation on the quotient A^β.

(4) An element x of a monoid M is *invertible* if and only if its image $R_x : M \to M$ under the right regular representation $R : M \to M^M$ is an invertible function.

   (a) Show that the set M^* of invertible elements of M forms a submonoid of M.

   (b) Prove $x \in M^* \Rightarrow \exists y \in M.\ xy = 1_M$.

(5) If A and B are finite subgroups of a group G, prove that

$$|AB| \cdot |A \cap B| = |A| \cdot |B|.$$