MATH 504 FALL 2003 MID-TERM

Hand in 3 answers for grading. Each question is worth $8\frac{1}{3}$ points.

(1) Suppose that a function $f : A \to B$ has non-empty domain. Show that there is a function $g : B \to A$ with $fgf = f$.

(2) Consider the following statement:

> If A and B are finite sets, and there is a surjection $f : A \to B$, then $|A| \geq |B|$.

Give a careful proof of the statement by induction on $|B|$.

(3) Let $\{\alpha_i \mid i \in I\}$ be a set of equivalence relations on a set A. Prove that the intersection $\bigcap_{i \in I} \alpha_i$ is also an equivalence relation on A.

(4) Let M be a monoid. Show that the power set 2^M becomes an M-set under the action

$$2^M \times M \to 2^M; (Y, m) \mapsto \{u \in M \mid mu \in Y\}.$$

(5) If A and B are finite subgroups of a group G, prove that

$$|AB| \cdot |A \cap B| = |A| \cdot |B|.$$