MATH 307D FALL 2000 TEST #5

Write clearly. Box or underline your final answers to computational questions.

(1) (5 points) Find the coordinate vector of the standard basis vector $\mathbf{e}_2$ with respect to the basis

$$\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\2\\3\end{bmatrix}, \begin{bmatrix}0\\0\\1\\3\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

of $\mathbb{R}^4$.

(2) (7 points) Sketch the level curve defined by the equation

$$x_1^2 + 6x_1x_2 + x_2^2 = 1.$$

Draw and label the principal axes, label the intercepts of the curve with the principal axes, and give the formula of the curve in the coordinate system defined by the principal axes.

(3) (8 points) Find closed formulas for the components of the dynamical system

$$\mathbf{x}(t+1) = \begin{bmatrix}1 & 0\\-1 & 2\end{bmatrix}\mathbf{x}(t)$$

with initial value $\mathbf{x}_0 = \mathbf{e_1}$.