MATH 307D FALL 2000 TEST #4

Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.

1. Consider a 3×3 matrix A with rows $\mathbf{x}, \mathbf{y}, \mathbf{z}$ in order from top to bottom. If $\det A = 10$, find:
   a: $\det(4A^T)$;
   b: $\det B$, where

$$B = \begin{bmatrix} \mathbf{y} - 2\mathbf{z} \\ \mathbf{x} + \mathbf{y} - \mathbf{z} \\ \mathbf{x} \end{bmatrix}.$$

2. For the matrix

$$A = \begin{bmatrix} -5 & 3 & 0 \\ -6 & 4 & 2 \\ 2 & -1 & 1 \end{bmatrix},$$

   find an invertible matrix S and a diagonal matrix D such that $S^{-1}AS = D$.

3. (a) Let A be an orthogonal matrix. Show that $\det A = 1$ or $\det A = -1$.
   (b) Give an example of a 2×2 orthogonal matrix A with $\det A = -1$.

4. For the matrix

$$A = \begin{bmatrix} 7 & 6 \\ 6 & 2 \end{bmatrix},$$

   find an orthogonal matrix S and a diagonal matrix D such that $S^{-1}AS = D$.