MATH 307D FALL 2000 TEST #1

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

1. Find all solutions to the following system of linear equations:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= -2 \\ x_1 \qquad\qquad\qquad + x_5 &= 4 \\ x_1 + x_2 \qquad\qquad\qquad &= 5. \end{aligned}$$

2. Determine whether the following matrix A is invertible:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

If it is, find A^{-1}.

3. A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$$T(\mathbf{x}) = 2(\mathbf{u} \cdot \mathbf{x})\mathbf{u} - \mathbf{x},$$

where $\mathbf{u}$ is the vector

$$\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}.$$

Find the matrix A of the transformation T.

4. Suppose that $\mathbf{x}_p$ is a solution of the linear system $A\mathbf{x} = \mathbf{b}$. Suppose that $\mathbf{x}_g$ is a solution of the "homogeneous" system $A\mathbf{x} = \mathbf{0}$. Let k be a real number. Show that $\mathbf{x}_p + k\mathbf{x}_g$ is a solution of the system $A\mathbf{x} = \mathbf{b}$.