MATH 302 SPRING 2007 PRACTICE TEST #2

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

1. Find a monic polynomial $p(X)$ in $\mathbb{Q}[X]$ such that the quotient field $\mathbb{Q}[X]/p(X)\mathbb{Q}[X]$ is isomorphic to the field $\mathbb{Q}\left(\sqrt{1+i\sqrt{3}}\right)$.

2. Show that the fields
$$\mathbb{Q}\left(\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\right)$$
and
$$\mathbb{Q}\left(\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}\right)$$
are isomorphic.

3. Find the smallest positive integer n for which the splitting field of the polynomial
$$X^2+6X+2$$
over $\mathbb{Q}$ is $\mathbb{Q}(\sqrt{n})$.

4. Let J be the ideal $(X^2+X+1)\mathbb{Z}/_5[X]$ of the ring $\mathbb{Z}/_5[X]$ of polynomials over the 5-element field $\mathbb{Z}/_5$. Find the inverse of the coset $X+J$ in the quotient field $\mathbb{Z}/_5[X]/J$.