

MATH 302 SPRING 2001 TEST #2

*Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.*

- (1) Write down an irreducible monic polynomial of degree 2 over \mathbb{Z}_5 . Explain why your polynomial is irreducible.
- (2) In $\mathbb{Q}[x]$, consider the polynomials $f(x) = x^3 - 3x^2 + 2x - 6$ and $g(x) = x^4 + 4x^2 + 4$. Express the greatest common divisor of $f(x)$ and $g(x)$ as a linear combination $u(x)f(x) + v(x)g(x)$.
- (3) Show that each element of the four-element field $\{0, 1, \alpha, 1 + \alpha\}$ is a root of the polynomial $x^4 - x$.
- (4) Find two zero divisors in the ring $\mathbb{Q}[x]/\langle x^3 - 8 \rangle$.