

MATH 302 SPRING 2001 TEST #1

Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.

- (1) State the axioms for a ring. Then give a careful proof, based on the axioms, that $(-a)(-b) = ab$ holds in each ring.
- (2) Prove that the rings \mathbb{Z}_4 and $\mathbb{Z}_2 \times \mathbb{Z}_2$ are not isomorphic.
- (3) Let

$$U = \left\{ \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} \mid x, y, z \text{ in } \mathbb{R} \right\}$$

be the ring of upper triangular 2×2 real matrices under the usual matrix addition and multiplication.

(a) Show that

$$J = \left\{ \begin{bmatrix} 0 & t \\ 0 & 0 \end{bmatrix} \mid t \text{ in } \mathbb{R} \right\}$$

is an ideal of U .

(b) Show that the quotient ring U/J is isomorphic to the direct product $\mathbb{R} \times \mathbb{R}$ of two copies of the ring \mathbb{R} of real numbers.

- (4) Let D be an ordered integral domain.

(a) Prove that

$$a^2 + b^2 \geq 2ab$$

holds for all a, b in D .

(b) Prove that $a^2 + b^2 = 2ab$ holds in D if and only if $a = b$.