

## MATH 302 SPRING 2001 FINAL

*Write clearly. Box or underline your final answers to computational questions.  
Each question is worth 5 points. Full credit is 40 points.*

- (1) Give a careful proof, quoting each ring axiom used at each step, that  $(-1)^2 = 1$  in a ring  $R$  with multiplicative identity 1.
- (2) Let  $\mathcal{C}$  be a collection of ideals of a ring  $R$ . Show that the intersection  $\bigcap_{J \in \mathcal{C}} J$  of the members of  $\mathcal{C}$  is itself an ideal of  $R$ .
- (3) Explain why  $\mathbb{Z}_{11}$  cannot be an ordered field.
- (4) A commutative ring  $K$  with multiplicative identity has no proper, non-trivial ideals. Show that  $K$  is a field.
- (5) Give an example of a commutative ring  $R$  that is not a field, even though it has no proper, non-trivial ideals.
- (6) Write down all 10 irreducible monic polynomials of degree 2 over  $\mathbb{Z}_5$ .
- (7) The map  $p : \mathbb{Z}_5 \rightarrow \mathbb{Z}_5; x \mapsto x^3 + 2$  is a permutation of  $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ . Write the permutation  $p$  of  $\mathbb{Z}_5$  as a product of disjoint cycles.
- (8) The graphs of two cubic functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  each pass through the points  $(0, 10)$ ,  $(1, 6.5)$ ,  $(2, 7.9)$  and  $(3, 8.1)$ . Prove that  $f = g$ .
- (9) Explain why the fields  $\mathbb{Q}(\sqrt{2})$  and  $\mathbb{Q}(\sqrt{3})$  are not isomorphic.
- (10) Explain why the fields  $\mathbb{Q}(e)$  and  $\mathbb{Q}(\pi)$  are isomorphic.
- (11) Find a parity check matrix for the linear binary code of length 5 generated by the codewords 10111 and 11100.