

MATH 301 SPRING 2008 PRACTICE FINAL

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

- (a) Determine the greatest common divisor d of 45 and 27.
(b) Express the greatest common divisor d as an integral linear combination of 45 and 27.
- Let x_1, x_2, \dots, x_{2m} be elements of a set X . In the symmetric group $X!$ on the set X , show that

$$\begin{aligned} & \left((x_1 \ x_2) \circ (x_3 \ x_4) \circ \cdots \circ (x_{2m-1} \ x_{2m}) \right)^{-1} \\ &= (x_{2m-1} \ x_{2m}) \circ \cdots \circ (x_3 \ x_4) \circ (x_1 \ x_2). \end{aligned}$$

- For $0 < \theta < \pi/2$, suppose that

$$\cos \theta = \frac{l}{n} \quad \text{and} \quad \sin \theta = \frac{m}{n}$$

with positive integers l, m , and n . Show that at least one of l and m is even.

- Consider the set

$$G = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \text{ in } \mathbb{Z}/5, ac = 1 \right\}$$

of matrices over the ring of integers modulo 5.

- (a) Show that G forms a group under multiplication.
(b) Show that $|G| = 20$.
- Give an example of subgroups H and K of a group G , such that HK is not a subgroup of G . Explain why HK is not a subgroup.
- Find a solution x to the simultaneous congruences

$$\begin{aligned} x &\equiv 3 \pmod{5}, \\ x &\equiv 7 \pmod{12}. \end{aligned}$$

- Let I and J be ideals of a ring R , with I a subset of J .
(a) Show that the quotient ring J/I is an ideal of R/I .
(b) Show that the quotient rings R/J and $(R/I)/(J/I)$ are isomorphic.