

MATH 301 SPRING 2008 PRACTICE TEST #2

Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.

- (1) Consider the set G of matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

satisfying the following three conditions:

- (a) a, b, c, d are integers;
(b)

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = 1;$$

- (c) a and d are odd, while b and c are even.

Determine whether G is or is not a subgroup of the general linear group $\text{GL}(2, \mathbb{R})$ of all invertible matrices.

- (2) Let $f : (X, \cdot, e_X) \rightarrow (Y, \cdot, e_Y)$ be a group homomorphism. Let N be a normal subgroup of X . Consider

$$f(N) = \{f(n) \mid n \text{ in } N\}.$$

- (a) If $f : X \rightarrow Y$ is surjective, show that $f(N)$ is a normal subgroup of Y .
(b) Give a counterexample to show that $f(N)$ need not be a normal subgroup of Y , if $f : X \rightarrow Y$ is not surjective.

- (3) Let H be a subgroup of a group G . Let N be a normal subgroup of G . Show that HN is a subgroup of G .
(4) Determine the group $(\mathbb{Z}/12)^*$ of units of the monoid $(\mathbb{Z}/12, \cdot, 1)$ of integers modulo 12 under multiplication.