

MATH 301 SPRING 2008 PRACTICE TEST #1

Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.

- (1) Let $f : X \rightarrow Y; x \mapsto f(x)$ be a function.
(a) Show that there is a subset Y' of Y such that

$$g : X \rightarrow Y'; x \mapsto f(x)$$

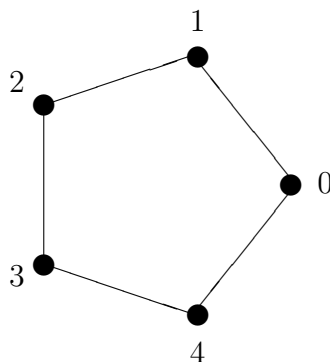
is surjective.

- (b) Show that there is a subset X' of X such that

$$h : X' \rightarrow Y'; x \mapsto f(x)$$

is bijective.

- (2) Write $\sigma_a : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto a + x$ for the shift by a real number a .
Suppose that a group G of permutations of \mathbb{R} contains σ_a and σ_b for real numbers a and b .
(a) Show that G contains σ_{ma} for each positive integer m .
(b) Show that G contains σ_{ma} for each integer m .
(c) Show that the group G contains σ_{ma+nb} for each integral linear combination $ma + nb$ of a and b .
- (3) Write each of the 10 symmetries of the regular pentagon



in 3-space as a product of disjoint cycles.

- (4) Which of the following three conditions determines the kernel relation R of the cosine function $\cos : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto \cos x$?
- (a) $x R y \Leftrightarrow x = \pm y$.
(b) $x R y \Leftrightarrow x = 2\pi n \pm y$ for some integer n .
(c) $x R y \Leftrightarrow x - y = 2\pi n$ for some integer n .