MATH 301 SPRING 2017 GRADED HOMEWORK #3

Write clearly. Credit is given for the best three answers.

(1) Let e be an element of a group $(G, \cdot)$.
   (a) Show that the set G forms a group under the multiplication
   $$* : G \times G \to G; (x, y) \mapsto xe^{-1}y.$$
   (b) Show that the group $(G, *)$ with multiplication given in (a) is isomorphic with the original group structure $(G, \cdot)$ on G.

(2) (a) Let x be an element of finite order in a group G. Show that the order of each power of x is a divisor of the order of x.
   (b) Let x be an element of odd finite order in a group G. Show that x^2 has the same order as x.

(3) Consider the subset $S = \{0, 2, 4, 6, 8, 10, 12\}$ of $\mathbb{Z}/14\mathbb{Z}$.
   (a) Show that $S = \{0, 2, 4, 6, 8, 10, 12\}$ forms a subring of $\mathbb{Z}/14\mathbb{Z}$.
   (b) Show that S forms a unital ring.
   (c) Show that S is not a unital subring of $\mathbb{Z}/14\mathbb{Z}$.

(4) Determine the group of units of the monoid of Gaussian integers under multiplication.