MATH 301 SPRING 2017 GRADED HOMEWORK #2

Write clearly. Credit is given for the best three answers.

(1) Let R be a reflexive and transitive relation on a set X. Define a new relation E on S by

$$x \; E \; y \quad \Leftrightarrow \quad x \; R \; y \text{ and } y \; R \; x$$

for $x, y \in X$. Show that E is an equivalence relation on X.

(2) Let $d_r d_{r-1} \ldots d_2 d_1 d_0$ be the usual decimal representation of a positive integer n. Show that n is divisible by 11 if and only if the condition

$$d_0 + d_2 + d_4 + \ldots \equiv d_1 + d_3 + d_5 + \ldots \mod 11$$

holds.

(3) Define

$$S = \left\{ \begin{bmatrix} n & m \\ 0 & n \end{bmatrix} \;\middle|\; m, n \in \mathbb{Z}\,,\; n \neq 0 \right\}.$$

Show S forms a commutative monoid under the usual matrix multiplication.

(4) Suppose that elements a, b, c, d of a group $(G, \cdot, e)$ satisfy the equation $abcd = e$. Give a careful proof that $cdab = e$.