

MATH 301 SPRING 2017 GRADED HOMEWORK #1

Write clearly. Credit is given for the best three answers.

- (1) Let a be a natural number. Let d be a positive integer. Suppose that $a = dq + r$ is the result of applying the Division Algorithm with a as dividend and d as divisor. Show that

$$q = \max\{x \in \mathbb{N} \mid d \cdot x \leq a\}.$$

- (2) Give an example of a prime number p such that the number $2^p - 1$ is composite. Justify your answer.
- (3) Let X be a non-empty set. Let $f: X \rightarrow X$ be a function. Show that there is a function $g: X \rightarrow X$ such that $f = f \circ g \circ f$.
- (4) Consider the set

$$M = \{f: n \mapsto 3^r n + s \mid r, s \in \mathbb{N}\}$$

of functions on the base set \mathbb{N} . Show that M is a monoid of functions on \mathbb{N} .