

MATH 301 SPRING 2017 GRADED HOMEWORK #0

Write clearly. Credit is given for the best three answers.

- (1) There are 36 inches in a yard, and 100 centimeters in a meter.
 - (a) In how many ways can a piece of wood a yard long be divided into equal pieces whose length is an integral number of inches?
 - (b) In how many ways can a piece of wood a meter long be divided into equal pieces whose length is an integral number of centimeters?
- (2) Let c be a positive common divisor of two nonzero integers a and b . Show that $\gcd(a, b)/c = \gcd(a/c, b/c)$. You may assume that c divides $\gcd(a, b)$. You may not assume the Fundamental Theorem of Arithmetic.
- (3) Without using the Euclidean Algorithm, give a direct proof that for nonzero integers a, b , the greatest common divisor $\gcd(a, b)$ may be expressed as an integral linear combination of a and b . [Hint: Using the Well-Ordering Principle, show that $\gcd(a, b)$ is the smallest member of the set S of positive, integral linear combinations of a and b .]
- (4) Show that $4n + 2$ and $3n + 1$ are relatively prime for all even natural numbers n .