

MATH 301B SPRING 2016 GRADED HOMEWORK #3

Write clearly. Credit is given for the best three answers.

- (1) Let x be an element of odd finite order in a group G . Show that x^2 has the same order as x .
- (2) Consider the subset $S = \{0, 2, 4, 6, 8\}$ of $\mathbb{Z}/10\mathbb{Z}$.
 - (a) Show that $S = \{0, 2, 4, 6, 8\}$ forms a subring of $\mathbb{Z}/10\mathbb{Z}$.
 - (b) Show that S forms a unital ring.
 - (c) Show that S is not a unital subring of $\mathbb{Z}/10\mathbb{Z}$.
- (3) In a ring R , define $\widehat{R} = \{(r, r) \mid r \in R\}$. Show that R is a zero ring if and only if \widehat{R} is an ideal in $R \times R$.
- (4) Let n be an integer. If n leaves a remainder of 1 when divided by 3, and a remainder of 3 when divided by 13, what remainder does it leave when divided by 39?