

MATH 301B SPRING 2016 GRADED HOMEWORK #2

Write clearly. Credit is given for the best three answers.

- (1) Consider the numbers

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10.$$

Is it possible to put positive or negative signs in front of each, so that the total sum of the signed numbers is zero? Justify your answer.

- (2) Consider an angle θ with $0 < \theta < \frac{\pi}{2}$. Suppose that

$$\cos \theta = \frac{l}{n} \quad \text{and} \quad \sin \theta = \frac{m}{n}$$

are rational numbers, with positive integers l , m , and n .

- (a) Give a justified example of an angle θ with these properties.
(b) Show that l and m cannot both be odd numbers.
- (3) Let $\mathcal{P}_{\text{fin}}(\mathbb{N})$ denote the set of finite subsets of \mathbb{N} .
(a) Show that $\mathcal{P}_{\text{fin}}(\mathbb{N})$ forms a monoid under set union.
(b) Show that, under set intersection, $\mathcal{P}_{\text{fin}}(\mathbb{N})$ does form a semigroup.
(c) Show that, under set intersection, $\mathcal{P}_{\text{fin}}(\mathbb{N})$ does not form a monoid.
- (4) Let $u_1, u_2, \dots, u_{r-1}, u_r$ be elements of a group G . Give a careful proof, by induction on r , that

$$(u_1 u_2 \dots u_{r-1} u_r)^{-1} = u_r^{-1} u_{r-1}^{-1} \dots u_2^{-1} u_1^{-1}.$$