MATH 301B SPRING 2016 GRADED HOMEWORK #1

Write clearly. Credit is given for the best three answers.

(1) Show that 1 is the only positive integer that is relatively prime to each positive integer.

(2) Define $f(n) = 2^n - 1$ for each positive integer n.
  (a) If n is composite, show that $f(n)$ is composite.
  (b) If p is prime, show that $f(p)$ may be composite.

(3) Let X be a non-empty set. Let J be the set of injective self-maps on X, and let S the set of surjective self-maps on X.
  (a) Show that J is a monoid of functions on X.
  (b) Show that S is a monoid of functions on X.
  (c) Prove that $\forall\ s \in S\,,\ \exists\ j \in J\,.\ s \circ j = \mathrm{id}_X$.

(4) Let M be a monoid of functions on a base set X. Define

$$S = \{f\colon X \to X \mid \exists\ m \in M\,.\ f \circ m = \mathrm{id}_X\}\,.$$

Show that S is a monoid of functions on X.