MATH 301B SPRING 2016 GRADED HOMEWORK #0

Write clearly. Credit is given for the best three answers.

(1) Let $d > 1$ be a fixed integer, known as the *base*. To represent a given positive integer n as a sequence $n = n_k n_{k-1} \dots n_2 n_1$ of digits in base d, with $0 \leq n_i < d$ for $1 \leq i \leq k$, consider the following algorithm:
  (a) Initialize with $q_0 = n$ and $i = 1$;
  (b) At Step (i), obtain $q_{i-1} = q_i d + n_i$ with the Division Algorithm;
  (c) Stop at Step (k) when $q_k = 0$;
  (d) Otherwise, replace i by $i + 1$ and return to (b).

Show that $n = n_k d^{k-1} + n_{k-1} d^{k-2} + \dots + n_2 d + n_1$.

(2) Express the base 10 number 4195 as a *hexadecimal* (base 16) number. Use $A = 10$, $B = 11$, $C = 12$, $D = 13$, $E = 14$, $F = 15$ for the digits above 9.

(3) Without using the Euclidean Algorithm, give a direct proof that for nonzero integers a, b, the greatest common divisor $\gcd(a, b)$ may be expressed as an integral linear combination of a and b. [Hint: Using the Well-Ordering Principle, show that $\gcd(a, b)$ is the smallest member of the set S of positive, integral linear combinations of a and b.]

(4) Show that $\gcd(4n + 1, 5n + 1) = 1$ for all natural numbers n.