

MATH 301B SPRING 2010 PRACTICE TEST #1

*Write clearly. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Write $\sigma_a : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto a + x$ for the shift by a real number a . Suppose that a group G of permutations of \mathbb{R} contains σ_a and σ_b for real numbers a and b .
- Show that G contains σ_{ma} for each positive integer m .
 - Show that G contains σ_{ma} for each integer m .
 - Show that the group G contains σ_{ma+nb} for each integral linear combination $ma + nb$ of a and b .
- (2) Let β , and $\alpha = (x_1 \ x_2 \ \dots \ x_{r-1} \ x_r)$, be permutations of a finite set X . Show that

$$\beta \circ \alpha \circ \beta^{-1} = (\beta(x_1) \ \beta(x_2) \ \dots \ \beta(x_{r-1}) \ \beta(x_r)).$$

- (3) Consider the figure



Writing each element as a product of disjoint cycles, determine

- the group of symmetries of the figure in 2-space and
- the group of symmetries of the figure in 3-space.

- (4) Which of the following three conditions determines the kernel relation R of the cosine function $\cos : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto \cos x$?
- $x R y$ if and only if $x = \pm y$.
 - $x R y$ if and only if $x = 2\pi n \pm y$ for some integer n .
 - $x R y$ if and only if $x - y = 2\pi n$ for some integer n .