MATH 301C FALL 2007 PRACTICE TEST #2

Write clearly. All questions carry equal weight.

(1) Determine the group of units within the monoid $(\mathbb{Z}/_{20}, \cdot, 1)$ of integers modulo 20 under multiplication.

(2) Let H and K be subgroups of a group G. Define a relation R on G by $g_1 \ R \ g_2$ if and only if $hg_1k = g_2$ for some h in H and k in K. Show that R is an equivalence relation on G.

(3) Let $(G, \cdot, e)$ be a finite group, and let p be an odd prime number. Consider the equation

$$x^p = e$$

for an element x of G. Show that the number of solutions x in G is odd.

(4) Let $\mathrm{SO}_2(\mathbb{R})$ denote the set of all orthogonal real 2×2 matrices A with $\det A = 1$.
   (a) Show that $\mathrm{SO}_2(\mathbb{R})$ forms a normal subgroup of the group $\mathrm{O}_2(\mathbb{R})$ of all orthogonal real 2×2 matrices under matrix multiplication.
   (b) Show that $\mathrm{SO}_2(\mathbb{R})$ has index 2 in $\mathrm{O}_2(\mathbb{R})$.