

MATH 301A FALL 2013 GRADED HOMEWORK #3

Write clearly. Credit is given for the best three answers.

- (1) Let $(A, +, 0)$ be an abelian group. On the direct product $\mathbb{Z} \times A$, define a componentwise group structure and a multiplication

$$(m, a) \cdot (n, b) = (mn, mb + na).$$

- (a) Show that $(\mathbb{Z} \times A, +, \cdot)$ forms a commutative ring.
(b) Is $(\mathbb{Z} \times A, +, \cdot)$ unital? Justify your answer.
- (2) Suppose that $x^2 = x$ for each element x of a ring R .
(a) Show that R has characteristic 2.
(b) Show that R is commutative.
- (3) Let u be a non-zero element of a simple, commutative, unital ring R . Show that u is a unit of the monoid $(R, \cdot, 1)$.
- (4) Let G be the group of invertible matrices in the ring $(\mathbb{Z}/2)_{\mathbb{Z}}^2$ of 2×2 matrices over the ring of integers modulo 2. Determine $|G|$.
- (5) For natural numbers a and b , show that $a\mathbb{Z} + b\mathbb{Z} = \gcd(a, b)\mathbb{Z}$.