

MATH 301A FALL 2013 GRADED HOMEWORK #2

Write clearly. Credit is given for the best three answers.

- (1) Without using a calculator or Fermat's Last Theorem, show that $3987^{12} + 4365^{12} \neq 4472^{12}$.
- (2) For elements a, b, c, d of a group (G, \cdot, e) , suppose that $abcd = e$. Give a careful proof that $cdab = e$.
- (3) Let F denote the set of finite subsets of \mathbb{Z} .
 - (a) Show that F forms a monoid under union.
 - (b) Show that F does not form a monoid under intersection.
 - (c) Show that F forms a group under

$$X + Y = (X \setminus Y) \cup (Y \setminus X).$$

- (4) A 2×2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is said to be *lower triangular* if $b = 0$.

- (a) Show that the set of lower triangular real 2×2 matrices forms a monoid L under matrix multiplication.
 - (b) Determine the group of units L^* of the monoid L .
- (5) Let X be a non-empty subset of a group G . Define $\langle X \rangle$ to be the intersection of all the subgroups of G that contain X . Show that $\langle X \rangle$ is a subgroup of G .