MATH 301A FALL 2013 GRADED HOMEWORK #1

Write clearly. Credit is given for the best three answers.

(1) Let n be a positive integer. Prove that the set

$$\{d \in \mathbb{N} \mid d \,\big|\, n\}$$

is finite.

(2) For each real number b and non-zero real number a, define

$$\alpha_{a,b} : \mathbb{R} \to \mathbb{R}; x \mapsto ax + b.$$

Give a careful proof by induction that

$$\alpha_{a,b}^n(x) = a^n x + b \sum_{r=0}^{n-1} a^r$$

for each natural number n.

(3) Let X be a non-empty set. Let J be the set of injective self-maps on X, and let S the set of surjective self-maps on X.

(a) Show that J is a monoid of functions on X.

(b) Show that S is a monoid of functions on X.

(c) Prove that $\forall\ j \in J\ ,\ \exists\ s \in S\ .\ s \circ j = \mathrm{id}_X$.

(4) Let M be a monoid of functions on a base set X. Define

$$S = \{f \colon X \to X \mid \exists\ m \in M\ .\ m \circ f = \mathrm{id}_X\}\ .$$

Show that S is a monoid of functions on X.

(5) Let G be the set of permutations α of the set $\{0, 1, 2, 3\}$ with $\alpha(0) < 2$. Determine whether G is a group of permutations, and justify your answer.