

MATH 301A FALL 2000 TEST #1

Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.

1. (a) Express the permutation

$$\pi = (1\ 4\ 2\ 5)(3\ 4\ 5\ 6)(1\ 5)$$

as a product of disjoint cycles.

- (b) Is the permutation π even or odd?

2. For each real number b and non-zero real number a , define

$$\alpha_{a,b} : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto ax + b.$$

Let $G = \{\alpha_{a,b} \mid b \in \mathbb{R}, 0 \neq a \in \mathbb{R}\}$ be the *affine group* of these maps under the operation of composition. Show that the subset

$$P = \{\alpha_{a,b} \mid b \in \mathbb{R}, 0 < a \in \mathbb{R}\}$$

of G is a subgroup of G .

3. (a) Let a, b, c be elements of a group G , with $abc = 1$. Give a careful proof that $cab = 1$.
(b) Give an example of elements a, b, c of a group G such that $abc = 1$, but $bac \neq 1$.
4. Show that the difference between the cubes of two successive integers is never divisible by three.