MATH 301A FALL 2000 FINAL

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

(1) Write out the Cayley table of a group on the set $\{o, p, q\}$ with o as the identity element.

(2) List all 8 elements of the subgroup of S_4 generated by the subset $\{(12)(34), (13)\}$. You must express each element as a product of disjoint cycles.

(3) Let M and N be normal subgroups of a group G. Show that the subset $MN = \{mn \mid m \in M, n \in N\}$ of G is a normal subgroup of G.

(4) Prove $\mathrm{Aut}(\mathbb{Z}_8) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.

(5) For a group G, let $\widehat{G}$ be the diagonal subset

$$\widehat{G} = \{(g, g) \mid g \in G\}$$

of $G \times G$. Show that the group G is abelian if and only if $\widehat{G}$ is a normal subgroup of $G \times G$.

(6) Show that there is no simple group of order 28.