MATH 3010-1 SPRING 2025 PRACTICE TEST #2

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.

(1) Determine the group of units $(\mathbb{Z}/_{20}, \cdot, 1)^*$ of the monoid $(\mathbb{Z}/_{20}, \cdot, 1)$ of integers modulo 20 under multiplication.

(2) Show that a group G is commutative if and only if the map

$$m\colon G \times G \to G; (x, y) \mapsto xy$$

is a semigroup homomorphism.

(3) Let G be a group, with normal subgroups H and K of coprime orders. Show that each element h of H commutes with each element k of K.

(4) Let a and b be positive integers. Show that $a\mathbb{Z} \cap b\mathbb{Z} = \operatorname{lcm}(a, b)\mathbb{Z}$.