MATH 3010-1 SPRING 2025 PRACTICE TEST #1

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

(1) Consider a function $f\colon X \to Y$ with nonempty domain X. Show that there is a function $g\colon Y \to X$ such that $f = f \circ g \circ f$.

(2) Let $\alpha = (x_1\ x_2\ \ldots\ x_{r-1}\ x_r)$ and $\beta = (y_1\ y_2\ \ldots\ y_{s-1}\ y_s)$ be permutations of a finite set X. If $\{x_1, \ldots, x_r\} \cap \{y_1, \ldots, y_s\}$ is empty, show that $\alpha \circ \beta = \beta \circ \alpha$.

(3) Give a careful proof to show that $11^{\frac{2}{3}}$ is irrational.