MATH 3010 SPRING 2025 GRADED HOMEWORK #3

Write clearly. Credit is given for the best three answers.

(1) Suppose that R is a reflexive and transitive relation on a set X. Define a new relation E on X by
$$x \; E \; y \quad \Leftrightarrow \quad x \; R \; y \text{ and } y \; R \; x$$
for $x, y \in X$. Show that E is an equivalence relation on X.

(2) Let $d_r d_{r-1} \dots d_2 d_1 d_0$ be the usual decimal representation of a positive integer n, so that $n = \sum_{k=0}^{r} d_k 10^k$ with $0 \leq d_k < 10$ for $0 \leq k \leq r$. Show that n is divisible by 11 if and only if the condition
$$d_0 + d_2 + d_4 + \dots \equiv d_1 + d_3 + d_5 + \dots \mod 11$$
holds.

(3) Define
$$S = \left\{ \begin{bmatrix} 2^n & m \\ 0 & 2^n \end{bmatrix} \;\middle|\; m \in \mathbb{Z}\,,\ n \in \mathbb{N} \right\}.$$
Show S forms a commutative monoid under the usual matrix multiplication.

(4) Suppose that elements a, b, c, d of a group $(G, \cdot, e)$ satisfy the equation $abcd = e$. Give a careful proof that $cdab = e$.