

MATH 201B SPRING 2010 PRACTICE FINAL

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.*

- (1) Let A , B , and C be subsets of a set U . Prove that
 $(C \setminus A) \cup (A \setminus B) \cup (B \setminus C) = (A \cup B \cup C) \setminus (A \cap B \cap C)$.
- (2) Let n be an integer. Prove that $5 \mid (n^4 - 1)$ implies $5 \nmid n$.
- (3) Show that $\log_3 \sqrt{5}$ is irrational.
- (4) Let n be a positive integer. Prove or disprove:

$$3 \mid (2^{2n} - 1).$$

- (5) Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

diverges to infinity.

- (6) (a) Give an ε - δ type proof that $\lim_{x \rightarrow 10} x = 10$.
(b) Use limit theorems and induction on n to show that x^n is continuous at $x = 10$ for each positive integer n .