35. The Harmonic Series.

Proposition. If
$$\sum_{n=h}^{\infty} x_n$$
converges, then $\lim x_n = 0$.

Proof. A convergent series is a Cauchy series, so
$$\forall\, \varepsilon > 0\,,\ \exists\, M \in \mathbb{N}\,.\ \forall\, M < k\,,\ |x_k| = \left|\sum_{n=h}^{k} x_n - \sum_{n=h}^{k-1} x_n\right| < \varepsilon. \quad \square$$

The converse is false:

The Harmonic Series. This is
$$\sum_{n=1}^{\infty} \frac{1}{n}\,.$$
Although its summands $1/n$ tend to zero, the series diverges by the Comparison Test:
$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{1}{n} &= 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \dots \\
&\geq 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \dots \\
&= 1 + \frac{1}{2} + \left(\frac{2}{4}\right) + \left(\frac{4}{8}\right) + \dots \\
&= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots\,,
\end{aligned}$$
which is unbounded, and thus divergent.