18. Tweaking the induction hypothesis

"$\forall\, n \in \mathbb{N}\,,\ P(n)$" may actually be "$\forall\, l \in \mathbb{N}\,,\ P(l+r)$", some $r \in \mathbb{Z}$. Then, the induction basis is $l = 0$, corresponding to $P(r)$. Thus:

Proposition. $\forall\, r \leq m \in \mathbb{Z}\,,\ P(m)$.

Proof. ... by induction on m.
Induction Basis: $P(r)$ holds.
Induction Step: $\forall\, r \leq m \in \mathbb{Z}\,,\ P(m) \Rightarrow P(m+1)$. □

In this case, the induction hypothesis $P(m)$ may be replaced by:

$$r \leq m \text{ and } P(m)$$

Strong induction. So-called "strong induction" to prove

Proposition: $\forall\, n \in \mathbb{N}\,,\ P(n)$

just means plain induction to prove the equivalent

Proposition: $\forall\, n \in \mathbb{N}\,,\ Q(n)$

with tweaked induction hyptothesis $Q(n) \equiv P(0) \wedge P(1) \wedge \ldots \wedge P(n)$.

The art of tweaking an induction hypothesis. You have to prove the induction step

$$P(n) \Rightarrow P(n+1)\,.$$

Consider strengthening $P(n)$.

Good news: With a more powerful hypothesis $P(n)$,
you have more to work with in the direct proof.
Bad news: With a more powerful hypothesis $P(n)$,
the conclusion $P(n+1)$ becomes harder to prove.