15. Disproof

True or false? To decide whether a statement is true or false, try doing some experiments to check.

In number theory: Try some sample numbers.
In set theory: Draw a Venn diagram.
For real variables: Use a graphing calculator (or sketch a graph).

Disproving an existence statement. To disprove $\exists\ x \in U\ .\ P(x)$, declare it "FALSE",
then formulate and prove its universally quantified negation:

Proposition. $\forall\ x \in U\ ,\ \neg P(x)$.

Disproving a universally quantified statement. Disproving $\forall\ x \in U\ ,\ P(x)$ means giving a constructive proof of its existentially quantified negation: $\exists\ x \in U\ .\ \neg P(x)$.
There is a special format for this.

Counterexamples. First declare the statement "FALSE",
then present the **counterexample** x, showing $x \in U$ and $\neg P(x)$.

Counterexample should be concrete, and as simple as possible.

Counterexample to $[\forall\ x \in U\ ,\ P(x) \Rightarrow Q(x)]$
is $x \in U$ with $P(x) \wedge \neg Q(x)$.