

13. EQUIVALENCE AND EXISTENCE PROOFS

Equivalence proofs. Break up into separate directions:

Proposition. $P \Leftrightarrow Q$.

Proof. $(\Rightarrow) P = P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow \dots \rightarrow P_{m-1} \rightarrow P_m = Q$.

$(\Leftarrow) Q = Q_0 \rightarrow Q_1 \rightarrow Q_2 \rightarrow \dots \rightarrow Q_{n-1} \rightarrow Q_n = P$. □

In some easy cases, do both directions at once.

Proposition. $P \Leftrightarrow Q$.

Proof. $P = P_0 \leftrightarrow P_1 \leftrightarrow P_2 \leftrightarrow \dots \leftrightarrow P_{m-1} \leftrightarrow P_m = Q$. □

Existence proofs. For now, consider *constructive* existence proofs.

Proposition. $\exists x \in U . P(x)$.

Proof. Present element x in U , and show that x has property $P(x)$. □