11. Contrapositive proof

Converse and contrapositive.

Original: $P \to Q$
Converse: $Q \to P$, no general logical relation to the original
Contrapositive: $(\neg Q) \to (\neg P)$

Have logical equivalence $[(\neg Q) \to (\neg P)]] \equiv (P \to Q)$
between original $P \to Q$ and contrapositive $(\neg Q) \to (\neg P)$
— both false just when P is true and Q is false.

Direct proof or contrapositive proof? In $P \to Q$,
which is more complicated, hypothesis P or conclusion Q?

DIRECT PROOF:

Proposition. Simple hypothesis P implies complicated conclusion Q.

Proof. $P = P_0 \to P_1 \to P_2 \to \ldots \to P_{n-1} \to P_n = Q$. □

CONTRAPOSITIVE PROOF:

Proposition. Complicated hypothesis P implies simple conclusion Q.

Proof. $(\neg Q) = Q_0 \to Q_1 \to Q_2 \to \ldots \to Q_{n-1} \to Q_n = (\neg P)$. □