10. Proof by cases

Proposition. Hypothesis P implies conclusion Q.

Proof. **Case A:** In Case A,
$P = P_0 \to P_1 \to P_2 \to \ldots \to P_{n-1} \to P_n = Q$.
[End of Case A announced by start of Case B.]
Case B: In Case B,
$P = P_0 \to P_1 \to P_2 \to \ldots \to P_{n-1} \to P_n = Q$.
$\vdots$
Case Z: In Case Z,
$P = P_0 \to P_1 \to P_2 \to \ldots \to P_{n-1} \to P_n = Q$. □

Proposition. Hypothesis P implies conclusion Q.

Proof. **Case A:** In Case A,
$P = P_0 \to P_1 \to P_2 \to \ldots \to P_{n-1} \to P_n = Q$.
[End of Case A announced by start of Case B.]
Case B: Similar to Case A. □

or

Proof. Without loss of generality, take Case A,
$P = P_0 \to P_1 \to P_2 \to \ldots \to P_{n-1} \to P_n = Q$. □

Definition. Consider $m, n \in \mathbb{Z}$.

(1) The integers m and n *have the same parity* if they are both odd, or both even,
(2) The integers m and n *have opposite parity* if one is odd, and the other is even.