## 9. Direct proof

**Proposition.** Premise or <u>hypothesis</u> $P$ implies <u>conclusion</u> $Q$.
Formally, $P \to Q$.

*Proof.* $P = P_0 \to P_1 \to P_2 \to \ldots \to P_{n-1} \to P_n = Q$. □

**Working with definitions.** To start $(P = P_0 \to P_1)$
or finish $(P_{n-1} \to P_n = Q)$ a direct proof,
work with the definitions of the terms
appearing in the hypothesis $P$ or conclusion $Q$.

WARNING: In a definition, "if" means "if and only if"!

**Definition.** Consider $n \in \mathbb{Z}$.

(1) The integer $n$ is *even* if it can be written
in the form $n = 2r$ for some integer $r$.
(2) The integer $n$ is *odd* if it can be written
in the form $n = 2r + 1$ for some integer $r$.

**Definition.** Consider $d, m \in \mathbb{Z}$.

(1) The integer $m$ is a *multiple* of $d$ if it can be written
in the form $m = dr$ for some integer $r$.
(2) The integer $d$ *divides*, or is a *divisor* of $m$,
if $m$ is a multiple of $d$.

**Notation.** For integers $d$ and $m$,
the notation $d \mid m$ means "$d$ divides $m$", which is an open statement.
Don't confuse with $d/m$, which is a rational number (assuming $m \neq 0$).