8. Negating statements

Negating quantified statements. Remember:

Flip the quantifier(s), negate the punchline.

$$\neg[\ \forall\, x \in U\,,\ P(x)\ ] \qquad \neg[\ \exists\, x \in U\,.\ P(x)\ ]$$
$$\equiv\ \exists\, x \in U\,.\ \neg P(x) \qquad \equiv\ \forall\, x \in U\,,\ \neg P(x)$$

Negating natural language statements. translate/negate/translate

The real number x is positive and π is less than or equal to 7.
TRANSLATE $\searrow$
$(x > 0)$ and $(\pi \leq 7)$
NEGATE $\searrow$
$(x \leq 0)$ or $(\pi > 7)$
TRANSLATE $\searrow$
The real number x is nonpositive or π is greater than 7.

Negating implications. Use quantifiers.

If x is negative, then $x^3 \geq 0$.
TRANSLATE $\searrow$
$\forall\, x \in\,]-\infty, 0\,[\,,\ x^3 \geq 0$
NEGATE $\searrow$
$\exists\, x \in\,]-\infty, 0\,[\,.\ x^3 < 0$
TRANSLATE $\searrow$
There is a negative real number x with $x^3 < 0$.

Negating implications. Use logical equivalence.

$\neg[P \to Q] \equiv \neg[(\neg P) \vee Q] \equiv P \wedge (\neg Q)$