6. Logical equivalence

Checking with a truth table.

$$(P \leftrightarrow Q) \equiv (P \to Q) \land (Q \to P)$$

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \land (Q \to P)$	$P \leftrightarrow Q$
F	F	T	T	T	T
F	T	T	F	F	F
T	F	F	T	F	F
T	T	T	T	T	T

Algebra laws.

De Morgan: $\neg(P \land Q) \equiv (\neg P) \lor (\neg Q)$, $\neg(P \lor Q) \equiv (\neg P) \land (\neg Q)$
Idempotent: $P \land P \equiv P$, $P \lor P \equiv P$
Distributive: $P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$
$P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$
Associative: $P \land (Q \land R) \equiv (P \land Q) \land R \equiv P \land Q \land R$
$P \lor (Q \lor R) \equiv (P \lor Q) \lor R \equiv P \lor Q \lor R$
Commutative: $P \land Q \equiv Q \land P$, $P \lor Q \equiv Q \lor P$

Using algebra laws.

$$\begin{aligned} \neg(P \land Q \land R) &\equiv \neg[(P \land Q) \land R] && \text{(associative law)} \\ &\equiv \neg(P \land Q) \lor (\neg R) && \text{(De Morgan law)} \\ &\equiv [(\neg P) \lor (\neg Q)] \lor (\neg R) && \text{(De Morgan law)} \\ &\equiv (\neg P) \lor (\neg Q) \lor (\neg R) && \text{(associative law)} \end{aligned}$$

gives "super De Morgan" $\neg(P \land Q \land R) \equiv (\neg P) \lor (\neg Q) \lor (\neg R)$