6. Logical equivalence

Checking with a truth table.

$$(P \leftrightarrow Q) \equiv (P \to Q) \wedge (Q \to P)$$

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$	$P \leftrightarrow Q$
F	F	T	T	T	T
F	T	T	F	F	F
T	F	F	T	F	F
T	T	T	T	T	T

Algebra laws.

De Morgan: $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)\,,\ \neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$
Idempotent: $P \wedge P \equiv P\,,\ P \vee P \equiv P$
Distributive: $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
Associative: $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R) \equiv P \wedge Q \wedge R)$
$P \vee (Q \vee R) \equiv (P \vee Q) \vee R \equiv P \vee Q \vee R$
Commutative: $P \wedge Q \equiv Q \wedge P\,,\ P \vee Q \equiv Q \vee P$

Using algebra laws.

$$\begin{aligned}
\neg(P \wedge Q \wedge R) &\equiv \neg[(P \wedge Q) \wedge R] && \text{(associative law)}\\
&\equiv \neg(P \wedge Q) \vee (\neg R) && \text{(De Morgan law)}\\
&\equiv [(\neg P) \vee (\neg Q)] \vee (\neg R) && \text{(De Morgan law)}\\
&\equiv (\neg P) \vee (\neg Q) \vee (\neg R) && \text{(associative law)}
\end{aligned}$$

gives “super De Morgan” $\neg(P \wedge Q \wedge R) \equiv (\neg P) \vee (\neg Q) \vee (\neg R)$